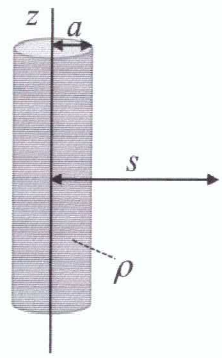
# EXAM 15-06-2015

ELECTRICITY AND MAGNETISM 1
#QUESTIONS: 4, #POINTS: 100

WRITE YOUR NAME AND STUDENT NUMBER ON EVERY SHEET. USE A SEPARATE SHEET FOR EACH PROBLEM. WRITE CLEARLY. USE OF A (GRAPHING) CALCULATOR IS ALLOWED. FOR ALL PROBLEMS YOU HAVE TO WRITE DOWN YOUR ARGUMENTS AND THE INTERMEDIATE STEPS IN YOUR CALCULATIONS.

## Question 1 - 25 points

A very long non-conducting cylinder of radius $a$ lies with its symmetry axis along the z-axis. It carries a uniform charge distribution $\rho$ (in $C/m^3$). Edge effects may be neglected.




**A.** Write down Gauss's law for the electric field in integral form.

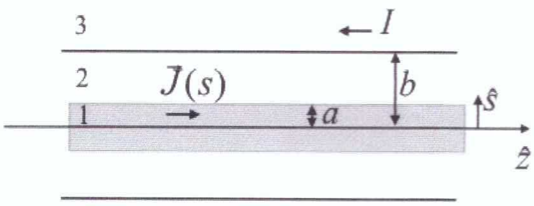
**B.** Find the electric field $\vec{E}$ *outside* the cylinder at a distance $s$ of the symmetry axis.

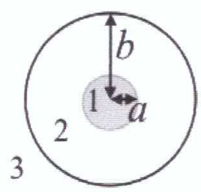
**C.** Suppose that the cylinder suddenly becomes conducting and all charge is allowed to move freely. How would that effect the electric field inside and outside the cylinder? Explain your answer.

Consider now that a cylinder that carries a current in the $+\hat{z}$ direction, given by the following current density:

$$\vec{J}(s) = \frac{a^2 J_0 e^{-\frac{s}{b}}}{2bs}\hat{z}$$

with $J_0$ a positive constant. A cylindrical surface of radius $b$ is coaxial with the solid cylinder. The cylindrical surface carries a current $I$ (in $C/s^{-1}$) in the $-\hat{z}$ direction. This current is uniformly distributed over the cylinder surface. Edge effects may be neglected.




**D.** The magnitude of the current $I_C$ through the solid cylinder with radius $a$ is equal to $I$. Show that

$$J_0 = \frac{I}{\pi a^2 (1 - e^{-\frac{a}{b}})}$$

**E.** We subdivide space into three regions, 1) $0 < s < a$; 2) $a \leq s \leq b$; 3) $s > b$. Find the magnetic field $\vec{B}$ in the regions 1, 2 and 3.

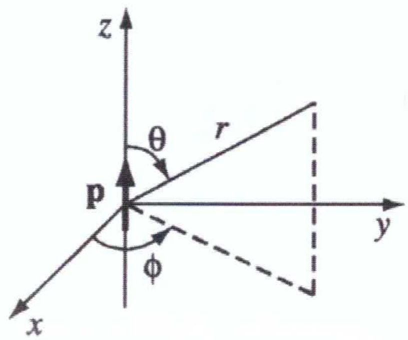
## Question 2 - 25 points

**A.** Sketch the electric field $\vec{E}$ in the (z,y) plane for
a) a perfect electric dipole;
b) a physical electric dipole,
with in both cases the dipole pointing in the $+\hat{z}$ direction.

**B.** Sketch the magnetic field $\vec{B}$ in the (z,y) plane for
a) a perfect magnetic dipole;
b) a physical magnetic dipole,
with in both cases the dipole pointing in the $+\hat{z}$ direction.

An electric dipole is located at the origin (0, 0, 0) (see figure). Its dipole moment is $\vec{p} = p\hat{z}$. If we assume that the dipole can be describes as a pure (or perfect) dipole, its potential is

$$V_{dip}(R,\theta) = \frac{\hat{r}\cdot\vec{p}}{4\pi\epsilon_0 r^2} = \frac{p\cos(\theta)}{4\pi\epsilon_0 r^2}.$$




**C.** Use this potential to show that the electric field of a pure dipole is

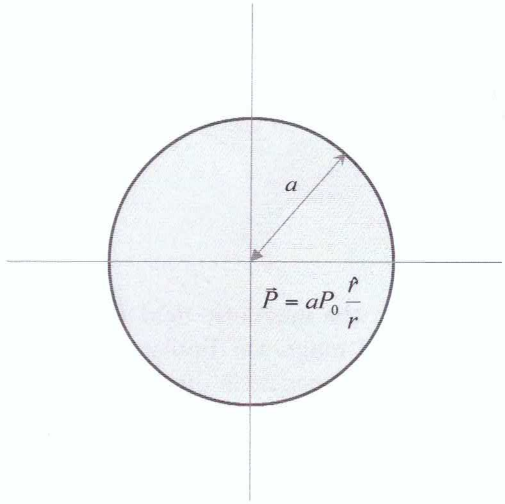
$$\vec{E}_{dip}(r,\theta) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos(\theta)\hat{r} + \sin(\theta)\hat{\theta})$$

**D.** Find the magnitude and direction of the force on a charge $q$ at the position (2a, 0, 0) on the x-axis. Give two expressions for this force, one in spherical and one in Cartesian coordinates.

**E.** If we were to add a charge $+q$ at the center of the coordinate system, is there a point at which the electric field of the monopole would exactly cancel that of the dipole? If yes, calculate the coordinates of that point. If not, argue why not.

## Question 3 - 25 points

Consider a solid sphere (of radius $a$) with its centre at the origin. The sphere carries a fixed polarization $\vec{P} = aP_0 \frac{\hat{r}}{r}$.

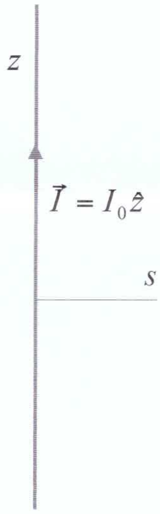



**A.** Find the bound surface charge density $\sigma_b$ at the surface of the sphere.

**B.** Find the bound volume charge density $\rho_b$ in the sphere.

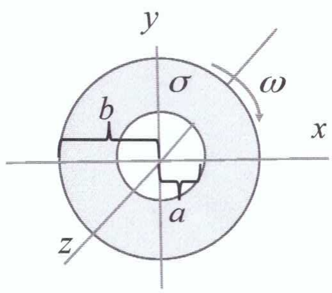
**C.** Show that the sphere is neutral.

An infinitely long wire lies along the z-axis and carries a current $\vec{I} = I_0\hat{z}$ (see figure).




**D.** Find the vector potential $\vec{A}$ at a distance $s$ to the wire. You may use your knowledge of the magnetic field of an infinite wire. Choose your own reference point at which the vector potential is zero.

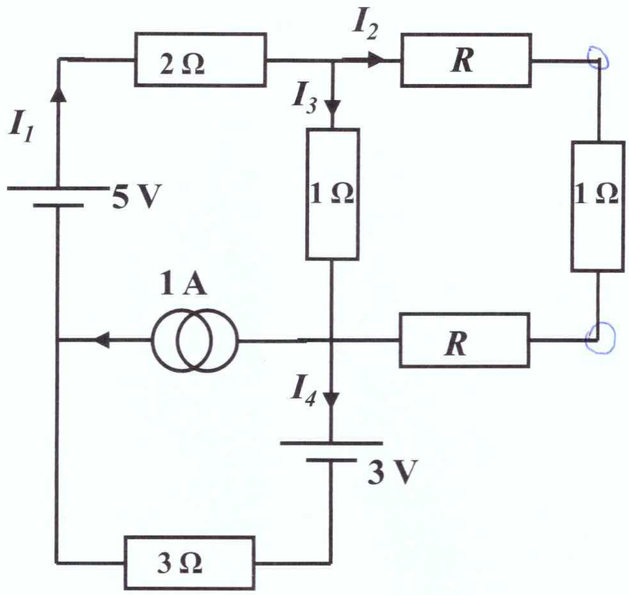
Consider a disk with surface charge density $\sigma$, inner radius $a$ and outer radius $b$ (see figure). The disk rotates clockwise around the z-axis with angular velocity $\omega$.




**E.** Find the magnetic dipole moment $\vec{m}$ of this disk.

## QUESTION 4 - 25 POINTS

Consider the circuit in the figure below.

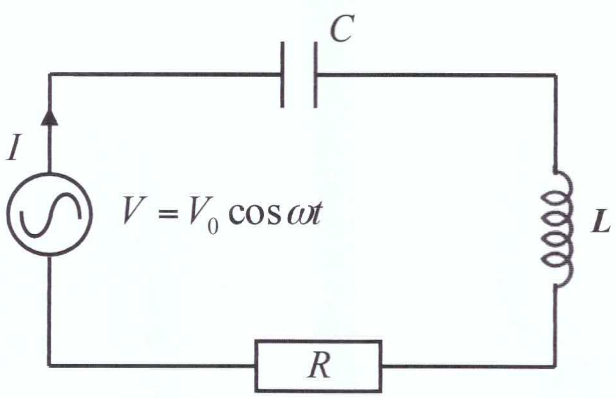



**A.** Find all the node equations (Kirchhoff 1). Show that one of these equations can be derived from the other equations.

**B.** Find all the loop equations (Kirchhoff 2).

**C.** Suppose $I_1 = 4I_2$. Find the value of the resistance $R$.

Consider the electric circuit in de figure below. The oscillating voltage source is described (in real representation) by $V = V_0 \cos(\omega t)$.




**D.** Calculate the time-dependent current $I$ in complex and in real notation.

**E.** Calculate the resonance frequency (in terms of L and C), and show that there is no phase difference between the current and the driving voltage at resonance.

THE END